A note about the Cambridge capital controversies:

What heterogeneous capital goods really mean.

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For discussion with Prof. T.K. Rymes

A Note about the Cambridge Capital Controversies: What heterogeneous Capital Goods Really Means.

I. What are the Cambridge capital controversies all about?

1. Is it an aggregation problem?
   a) a problem in the measure of capital?
   b) aggregation of production functions?
   c) is the problem due to heterogeneous capital goods?

2. Is it a conceptual problem?
   a) capital is not a factor such as land?
   b) technical progress is badly measured?
   c) capital is reproducible?
   d) production analysis vs exchange analysis?
   e) prices as scarcity indices or cost indices?
   f) shifts in the production function cannot be distinguished from moves along it?

3. Is it a methodological problem?
   a) logical vs historical Time?
   b) the irrelevance of comparative statics?
   c) the irrelevance of continuity?

All these problems are somehow interwined.
We shall focus on Joan Robinson's assertion that reswitching is unimportant, and her insistance on the irrelevance of the analysis of steady states for the real world [1975].

II. The main explanation for this statement of hers, seems to be the fact that different techniques necessarily require different capital goods. Therefore, changes in techniques can only be explained with a story about how the new capital goods are gradually substituted for the old ones. Since such a story is usually not being provided,

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comparative statics is said to be atemporal and useless. This is noted by J. Robinson in her very first criticism of neoclassical capital theory:

> "...Any change in the ratio of capital to labour involves a reorganization of methods of production and requires a change in the shapes, sizes and specifications of many or all of the goods appearing in the original list [1953-54 , p. 48].

> " In short, the comparison between equilibrium positions with different factor ratios cannot be used to analyse changes in the factor ratio taking place through time, and it is impossible to discuss changes(as opposed to differences) in neoclassical terms" [1953-54, p. 62-3].

She has persued the same idea afterwards:

> " Two different rates of profit, on either side of a switch point, represent two different techniques, each requiring a different bundle of specific physical inputs..." [1975a, p. 54].

> "Each time it passes a switch point ... there must be a certain period of investment and disinvestment installing the stock required for the latest technique and clearing away the debris of the former one" [1981, p. 137].

III. There is some confusion as to what "heterogeneous capital goods" really mean.

In the minds of neoclassical authors, it seems that heterogeneous capital goods can be associated to a two-sector model (one consumption good and one capital good) with several available techniques. Heterogeneity, in this case is due to the multiplicity of potential techniques.

This seems to be the case of Samuelson [1962]. He starts by assuming that there are a great variety of capital goods (alpha, beta, ...,999). He assumes that "any one capital good, call it alpha, looks entirely different from a second beta capital good. Thus think of one as a plow; another as a machine tool..." [1962, p. 218]. We are then told that alpha will be used in preference

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to beta depending on the level of the rate of profit [1962, p. 220]. We are then back to a two-sector model with multiple techniques. This Samuelson [1962, p. 225] calls "a quasi-realistic MIT model of diverse heterogeneous capital goods' processes".

On the other hand, myself and most neo-Ricardians would interpret "heteregeneous capital goods' model" as meaning a multi-sector model, with several different investment goods, with perhaps only one technique. Heterogeneity implies that there are at least three sectors, as in the A. Lowe model for instance, where one type of capital good is needed for the consumption sector whereas the second capital good sector produces machines that are utilized in the two investment sectors.

From what precedes, it should now be easily understood why neo-Ricardians show some uneasiness when they are told that capital controversies are about heterogeneous capital goods. Since re-switching, capital reversing and the like can be demonstrated within the simple two-sector model, where capital is homogeneous (within a technique), they do not see where capital heterogeneity sets in. On the other hand, Samuelson [1975] seems to understand very well the point made by J. Robinson in her article on the unimportance of re-switching. He goes at great lengths to show that if he wrote " y rises as x falls", he meant a <u>difference</u> rather than a <u>change</u>. Samuelson thus seems fully aware of Robinson's point, i.e. that two different techniques require different bundles of capital goods, when he writes that "no alchemist can turn one capital good into another" [1962, p. 218]. What is being argued here is that few students of the capital controversies have been aware of or have understood Robinson's point and its consequences.

IV. The case for different techniques being associated with different physical capital goods can be most easily demonstrated by using the usual elementary two-sector model.

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Let us assume the standard price equations, with subscript i standing for the investment sector and c for the consumption sector, and a given technique:

$$p_i = (r + d)\, p_i\, v_i + w\, u_i \qquad (1a)$$

$$p_c = (r + d)\, p_i\, v_c + w\, u_c \qquad (1b)$$

Solving for p_i, in terms of consumption goods ($p_c = 1$), and defining the index of mecanization $A = v_i\, u_c / u_c\, v_c$, we get:

$$p_i = u_i / \left[u_c + (r + d)\, u_i\, v_c\, (1 - A) \right] \qquad (2)$$

This is the price of capital goods as a function of the equilibrium rate of profit r, and depending on the technological parameters and on the rate of depreciation d. One can distinguish three cases:

(i) $dp_i / dr = 0$

This is the celebrated Samuelson case: With A = 1, the organic composition of capital is equal in both sectors, more specifically:

$$v_i / u_i = v_c / u_c \qquad (3)$$

The wage-profit frontier is a straight line and the price of capital is a <u>constant</u>, independent of the distribution between the wage rate w and the rate of profit.

$$p_i = u_i / u_c \qquad (4)$$

(ii) $dp_i / dr < 0$

This implies $A < 1$. The wage-profit frontier is a convex curve. The price of capital is <u>monotonically</u> decreasing with higher rates of profit. This is the so-called positive price Wicksell effect.

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(iii) $dp_i / d_r > 0$

We then have $A > 1$. The wage-profit frontier is a concave curve. The price of capital is monotonically increasing. This is the negative price Wicksell effect.

Let us can consider three techniques, alpha, beta, gamma, where each respectively fulfills our three cases (i), (ii), (iii), in this order From past analyses, we all know that there can be two swich-points between any two of these three techniques. This means that there are two wage-rates for which the equilibrium rate of profit is identical for the two techniques considered. Therefore, at those two switch-points, two distributive variables (wage and profit rates) and all prices which are common to both systems are identical [Garegnani 1970, p. 165]. In the particular instance of the two- sector model, there are only a consumption good and an investment good. What we shall now prove is that either the law of the unique price is false or different techniques require different capital goods (heterogeneous capital goods from technique to technique).

The proof follows from the derivative of equation(2). Since it does not change sign as we modify r, this means there can only be one rate of profit for which the price of capital p_i would be the same for alpha/beta or alpha/gamma, or beta/gamma. But there could be two rates of profit for which all common prices should be the same. Since the law of unique price cannot be disposed of, we must admit that the capital goods of two given techniques are not common. They are not the same.

This is clearly so when there are only alpha techniques (the Samuelson case), for equation (2) has been reduced to equation (4). The price of capital goods is generally not the same. There is however a class of cases for which equiproportionnal production coefficients yield equal capital prices. The sufficient and necessary requirement, in

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vector form, as can be seen from equation (3), is given by:

$$L_1 = b\ L_2 \qquad (5)$$

where $L_j = (u_i, u_c)$ of technique alpha of type j.[1].

In the very special case, alluded to by Samuelson [1962, p. 230] and which Ferguson [1972, p. 169] considers to be the real Samuelson case, where $u_i = u_c$ and $v_i = v_c$, this condition is always verified. But, of course, then, we are back to a one-sector model. The prices of the consumption and of the investment goods are equal.

V. Few students of capital controversies seem to have notice that the introduction of new techniques usually entail the introduction of new and different capital goods. G.C. Harcourt does not mention it in his book-length survey [1972, in particular p. 43]. However, he has two paragraphs about it in a later survey. "... Avec des w-r droites ..., même s'il ne se produit que de petits changements dans la valeur du capital cela n'empêche pas que peuvent y correspondre de profonds changements dans la composition des moyens de production; et à vrai dire, en général, ce sera le cas" [1976, p. 64]. He further refers to J. Robinson and Luigi Pasinetti.

For Pasinetti [1977, p. 165 fn .4] is quite explicit. He clearly indicates that in two-sector models, only consumption prices can be compared, for only consumption goods are common to two rival techniques.[2] Being aware of this phenomenon, it is not surprising that he would have underlined the lack of <u>continuity</u> among capital goods. There are two aspects to this lack of continuity. First, capital goods are different as we move from technique to technique; second, their value may change violently at switch points.

> "...Deux techniques peuvent parfaitement être aussi voisines que l'on veut dans l'échelle de variation du taux de profit et cependant les biens capitaux physiques qu'elles exigent peuvent être complètement différents... Le théorie du capital est un domaine impropre à l'application du calcul et de l'analyse infinité simaux, et donc à l'analyse marginaliste" [1969, p. 233].

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Luigi Spaventa has underlined the qualitative difference of capital goods in his discussion of the traverse, calling it the true problem of transition [1973, p. 163].

P. Garegnani, despite his stand about long-run equilibrium positions, and D. Harris are also aware of this point. For instance, Garegnani [1970, p. 147 fn 1] assigns different values to the depreciation parameter d of the capital good for each technique. So does Harris [1978, p. 123] in his book.

The latter also specifies that, in a two-sector model, different techniques entail different capital goods, "unless the production coefficients in the capital good sector remain the same" [p. 124]. This means that the u_i's and the v_i's are identical, whereas the u_c's and the v_c's are different. At a switch point, two techniques exhibiting these properties shall have the same price for capital goods, as can be seen from rewriting equation (2) under the form:

$$p_i = w\, u_i \left[1 - (r + d)\, v_i\right] \qquad (6)$$

This special case is of no use for the Samuelson case, since the technical coefficients are proportionnal. Furthermore in this (Harris) special case, there can be no reswitching, as a consequence of the derivatives of equation (2). There is only one point on the wage-profit frontier for which the price of capital goods is identical for both techniques, although these goods are the same.

Note however than when one takes into account what Joan Robinson calls the real value of capital, or what Samuelson calls a "gratuitous act of deflating by real wages" [1962, p. 228], identical capital goods evaluated at the same rate of profit would have the same "real price", regardless of the level of the real wage of the techniques in use. Let P_i be this real price. From equation (6) we get:

$$P_i = p_i / w = u_i / \left[1 - (r + d)\, v_i\right] \qquad (7)$$

It is then obvious that two techniques with identical technical coefficients in the capital good sector (the u_i' and v_i's) will always have

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the same real price of capital (and the same value of capital per worker), whether they are at a switch-point or not.

Pasinetti [1977, p. 165] points out that when new goods are introduced with different techniques, they can be incorporated by augmenting the matrices representing the different techniques. In this fashion, the vector of prices is identical for the two techniques, while the interindustry matrix includes extra rows or columns of zeroes and the vector of labor coefficient contains extra zeroes.

VII. I think that this is the error which S. Baldone has committed in his 1984 Cambridge Journal of Economics article. He assumes a multi-sector Samuelson case, i.e., the degree of mecanization is uniform in all sectors. He asserts the following.

Take two Samuelson techniques alpha and beta. Call p_a the price vector for all goods of technique alpha. Call k_a, the value of capital per worker in technique alpha; call L_a, the vector of labour coefficients for technique alpha, and similarly for beta. Then, at a switch point, we must have: $p_a = p_b$, and therefore:

$$(k_a (1 + r)/w + 1) L_a = [k_b (1 + r) /w + 1] L_b$$

Baldone says that these two techniques, if they have a switch point, cannot differ in only one productive process. They must differ by more. The reason is that k_a and k_b are scalars, whereas L_a and L_b are vectors which differ only by one component. The equality, he says, cannot be verified.

Baldone supposes that the consumption basket is fixed and he sets its weighed price equal to one. My answer would then be the following. If the different productive process concerns some consumption good, this does not matter since their weighted price is set equal to one. Now, if the different productive process affects a capital good, then from what we have seen, the unique different component in L_a and L_b concerns two physically different capital goods. It is thus not surprising that their price should not be equal. The vectors L_a and L_b should be

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expanded to incorporate this extra capital good. By expanding the Pa and the p_b vectors similarly, all components of price would be equal in both techniques, once they have been divided by their respective value of the price of the consumption good.

If I am right, then Baldone's analysis on p. 277-278 does not stand. This would be a good example of omitting that different techniques usually imply different capital goods.

VIII. Conclusions.

The attemps by P. Samuelson [1962] to rescue the vulgar aggregate neo-classical theory and to justify the empirical work of R. Solow and others was doomed to fail on two counts, both of which Samuelson himself seemed to be aware at the time. First, his results were based on the restrictive assumption of equal organic composition of capital. Second, he had to assume some mechanism to move from one form of capital good to the other as the optimal technique changed. Samuelson did not seem to realize at the time, however, that these two assumptions were useless and meaningless in describing real economies. As shown by A. Shaikh [1980] and H.A. Simon [1979], neo-classical production functions are statistical artifacts, measures without theory.

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Footnotes

1. An example:

Technique $\alpha 1$ has $u_i = 12$ $v_i = 9$
$u_c = 4$ $v_c = 3$

Therefore $p_i = 3$

Technique $\alpha 2$ has $u_i = 6$ $v_i = 18$
$u_c = 2$ $v_c = 6$

Therefore $p_i = 3$

The capital/labor ratios are however different:

$k_1 = 9/4$ $k_2 = 3/2$

2. He refers to Hicks [1965].

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References

S. Baldone, "From surrogate to pseudo production functions", Cambridge J. of Economics, Sept. 1984.

C. Ferguson, "The current state of capital theory: a tale of two paradigms", Southern Economic J., Oct. 1972.

P. Garegnani, "Heterogenous capital, the production function and the theory of distribution", Review of Economic Studies, July 1970é Quoted from the French version in Grellet (1976)

G.C. Harcourt, Some Cambridge controversies in the theory of capital, Cambridge U.P., 1972.

_____ , "Les controverses cambridgiennes après la tourmente", in Grellet (éd.), 1976. Slightly revised in Oxford Economic Papers, 1976.

D. Harris, Capital accumulation and income distribution, Stanford U.P., 1978.

J. Hicks, Capital and growth, Oxford U.P., 1965.

L. Pasinetti, "Switches of technique and rate of return in capital theory", Economic Journal, Sept. 1969. Quoted from the French version (1976).

_____ , Lectures on the theory of production, Columbia U.P., 1977.

J. Robinson, "The production function and the theory of capital", Review of Economic Studies, 1953-54.
Quoted from Harcourt and Laing (ed.), Capital and growth, Penguin, 1971.

_____ , "The unimportance of reswitching", Quaterly J. of Economics, Feb. 1975.

_____ , "Reswitching: reply", Q.J.E., Feb. 1975a.

_____ , What are the questions, M.E. Sharpe, 1981.

P. Samuelson, "Parable and realism in capital theory: the surrogate production function, Review of Economic Studies, June 1962. Quoted from Harcourt and Laing.

.../12

———— , "Steady-state and transient relations: a reply on reswitching", Quaterly J. of Economics, Feb. 1975.

A. Shaikh, "Laws of production and laws of algebra: Humbug II", in E.J. Nell (ed.)., Growth, profits and property, Cambridge U.P., 1980.

H.A. Simon, "On parsimonious explanations of production relations", Scandinavian J. of Economics, no. 4, 1979.

L. Spaventa, "Notes on problems of transition between techniques", in J.A. Mirlees and N.H.S. Stern (ed.)., Models of Economic growth, Macmillan, 1973.

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